
ANALYTICAL METHOD FOR THE ANALYSIS OF STONE-COLUMNS ACCORDING TO THE ROWE DILATANCY THEORY

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abstract

In this paper a new analytical method to analyse the behaviour of rigid foundations stabilized by end bearing stone-columns is proposed. The stone column and the surrounding soil are treated in axial symmetric conditions as a unit cell. The stone column is assumed to behave as an Mohr-Coulomb rigid-plastic material with non-associative flow rule according to the Rowe stress dilatancy theory and the soil as an elastic material. These common assumptions, combined with equilibrium and kinematic conditions, lead to the simple analytical closed-form solution for the prediction of the behaviour for rigid footings resting on stone-column reinforced ground. The parametric study is presented to show the effect of dilatancy of the granular material on the deformations and stresses in the ground and its beneficial effect on settlement reduction. The results of the new method are compared with some already known analytical methods and some published field test results and observations.

keywords

ground improvement, stone columns, ground settlements, dilatancy theory

1 INTRODUCTION

Stone columns or granular piles are frequently used for the stabilization of soft clays and silts and loose silty sands with large amount of fines. For low-rise buildings, highway facilities, storage tanks, embankments, bridge abutments and other structures that can tolerate some settlements, stone columns are one of the most frequently used methods of support due to the cost, effectiveness and ease of the installation. The beneficial effects of stone columns are increased stiffness, reduced settlements, increased time rate of settlements, increased shear strength and reduction of the liquefaction potential of soft ground.

The available methods for the estimation of the behaviour of foundations resting on soft soil stabilised by a large number of end-bearing stone-columns can be classified as either approximate methods with important simplifying assumptions or sophisticated methods based on elasticity and/or plasticity theory such as finite element model.

The majority of the proposed approximate methods assume infinitely wide, loaded area with end-bearing stone columns having constant diameter and spacing. For such loading and geometry conditions the stone column and the surrounding soil can be treated in axial symmetric conditions. This approach is commonly known as a unit cell concept and was adopted by several researchers.

Several approximate analytical solutions are available to estimate the settlement reduction of stabilized ground and stress concentration in the stone-columns. Many of them [1-3] are based on elastic approach considering the stone-column and the surrounding soil as elastic materials. However, elastic methods give the ratio between the vertical stress in the column and in the soil (also known as stress concentration factor) approximately equal to the ratio of constrained modulus of both materials. This ratio was found to be considerably higher than measured in the field and it is believed that elastic methods may easily overestimate the effects of stone columns on settlement reduction [4].

The elastic and elasto-plastic solutions presented by Balaam and Booker [2, 5] indicate that the problem can be idealized by assuming that the stone column is in a triaxial state and perhaps yielding, that there is no shear stress at the stone-soil interface, and that there is no yielding in the soil. These common assumptions have been implemented in many methods where stone column is considered to be in a state of plastic equilibrium and under a triaxial stress state [5-8]. In the majority of these methods it is assumed that when loaded, stone-column yields at constant volume. However, in the work by Van Impe and Madhav [8] the nonlinear analytical solution is presented, showing the beneficial effect of the stone-column dilation on the deformation behaviour of the stabilized ground.

The objective of this paper is to present an analytical closed-form elastic-rigid-plastic solution which takes into account confined yielding of the stone material according to the Rowe stress-dilatancy theory [9] and to show the beneficial effects of dilation on the settlement reduction.

2 METHOD OF ANALYSIS

If stone-columns are evenly distributed, a regularly shaped area around the stone-column may be consid-

ered as a “unit cell”, consisting of stone-column and the surrounding soft soil in a zone of influence (Fig. 1). To simplify the analysis the zone of influence is approximated by a circle with a diameter d_e equal to $1.05s$, $1.13s$ and $1.29s$, for triangular, square or hexagonal pattern, respectively, where s is the column spacing.

The ratio between the area of column A_c and the area of the zone of influence A_e is represented by the area replacement ratio A_r , defined as

$$A_r = \frac{A_c}{A_e} = \frac{d_c^2}{d_e^2} = \frac{r_c^2}{r_e^2} \quad (1)$$

Let us consider a unit cell on smooth rigid base loaded with uniform load through the smooth rigid raft. The high drainage capacity of the granular material ensures that it deforms under drained conditions. The immediate settlement of soil in undrained conditions is negligible compared to the total final settlements, and thus it will not be considered in the analysis [5].

It is assumed that the dense granular material in the column is in triaxial stress state, reaching its peak resistance and thus dilating. The self weight of the soil and the column is neglected, which is one of the main drawbacks of the proposed method.

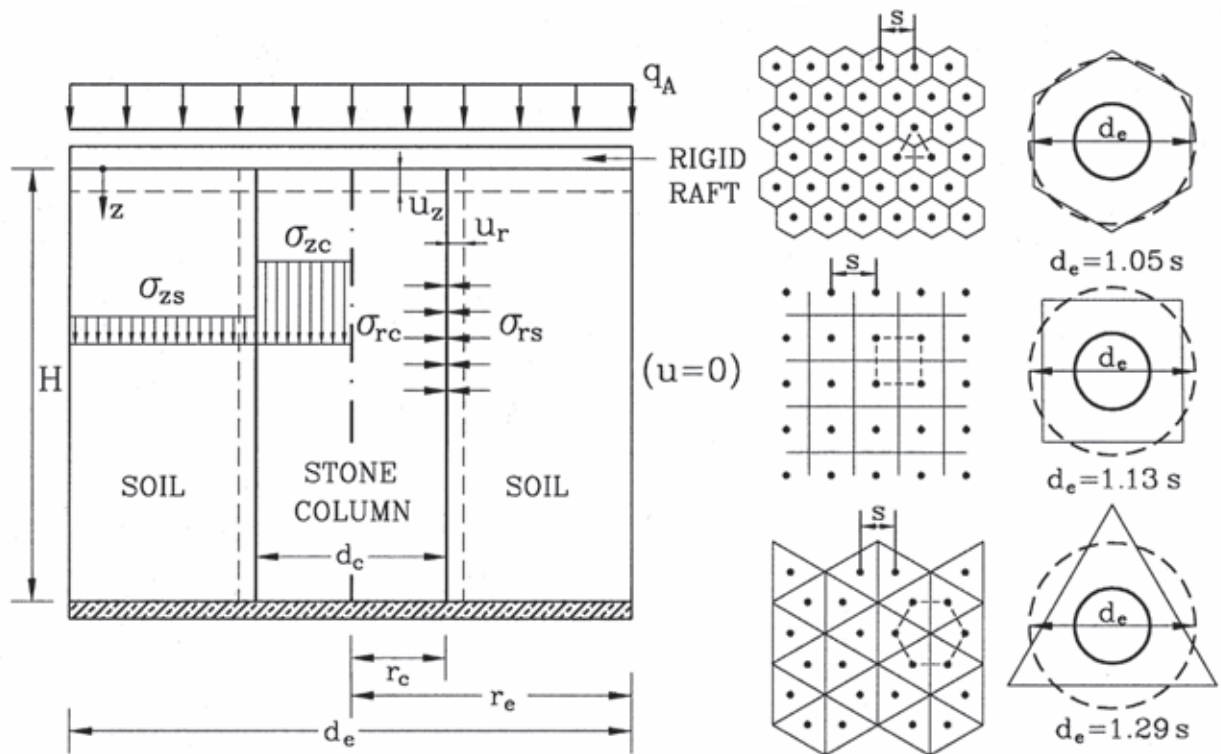


Figure 1. Basic features of the model based on regular patterns of stone-columns

Under uniform load q_A applied through rigid raft the end bearing stone-column and the surrounding soil will undergo the same vertical displacement u_z and radial displacement u_r , thus at the soil-column interface no slippage is expected between the soil and the granular material. The vertical, radial and volumetric strains of the stone-column are defined as

$$\begin{aligned} \varepsilon_z &= \frac{u_z}{H} & (2) \\ \varepsilon_r &= -\frac{u_r}{r_c} & (3) \\ \varepsilon_{vd} &= \varepsilon_z + 2\varepsilon_r & (4) \end{aligned}$$

where H is the height and r_c the radius of the column.

The relation between vertical and radial stress at the soil-column interface, σ_{zc} and σ_{rc} , in triaxial stress state can be simply obtained for the column material at yield:

$$\frac{\sigma_{zc}}{\sigma_{rc}} = \frac{1 + \sin \varphi'_c}{1 - \sin \varphi'_c} = K_{pc} \quad (5)$$

where φ'_c represents the peak triaxial shear angle of the column material. According to the Rowe stress dilatancy theory [9], Equation (5) can also be modified to:

$$\frac{\sigma_{zc}}{\sigma_{rc} \left(1 - \frac{\varepsilon_{vd}}{\varepsilon_z} \right)} = \frac{1 + \sin \varphi'_{cv}}{1 - \sin \varphi'_{cv}} \quad (6)$$

where φ'_{cv} is a triaxial shear angle of the stone material at constant volume. The relationship between the angle of dilatancy ψ and the peak friction angle φ'_c can be obtained using Rowe's equation [9]:

$$\sin \psi = \frac{\sin \varphi'_c - \sin \varphi'_{cv}}{1 - \sin \varphi'_c \sin \varphi'_{cv}} \quad (7)$$

Angle of dilatancy ψ can also be expressed in terms of the volumetric strain due to dilation and vertical strain of the column [10]:

$$\sin \psi = -\frac{\frac{\varepsilon_{vd}}{\varepsilon_z}}{2 - \frac{\varepsilon_{vd}}{\varepsilon_z}} = -\frac{\varepsilon_{vd}}{2\varepsilon_z - \varepsilon_{vd}} \quad (8)$$

The stress-strain behaviour of the column is entirely defined by Equations (5) and (8), and the two material parameters, which can be arbitrary selected between φ'_c , ψ and φ'_{cv} . The ratio between vertical and horizontal stresses in the column is defined by the strength of the

selected column material, while the ratio between the contained plastic strains of the stone-column material, ε_{vd} and ε_z , is determined by the selected value of the dilation angle ψ .

The soil surrounding the stone-column can be analysed as a thick cylinder using Equations (9) and (10) relating vertical and radial displacements, u_z and u_r , at the soil-column interface with vertical stress in the soil and radial interaction stress, σ_{zs} and σ_{rs} [11]:

$$u_z = \frac{H}{E_{oed}} \left[\frac{C_2 \sigma_{zs} - C_1 \sigma_{rs}}{C_3} \right] \quad (9)$$

$$u_r = \frac{r_c}{E_{oed}} \left[\frac{\sigma_{rs} - k_0 \sigma_{zs}}{C_3} \right] \quad (10)$$

where E_{oed} is the oedometric modulus of the soil and C_1 , C_2 and C_3 are constants defined as

$$C_1 = \frac{2k_0 A_r}{1 - A_r}, \quad C_2 = \frac{1 - 2\nu_s + A_r}{(1 - A_r)(1 - \nu_s)}, \quad C_3 = C_2 - k_0 C_1 \quad (11)$$

where ν_s is Poisson's ratio of the soil and $k_0 = \nu_s / (1 - \nu_s)$.

Applied vertical load q_A must be in equilibrium with vertical stresses in the column and in the soil:

$$q_A A_e = \sigma_{zc} A_c + \sigma_{zs} (A_e - A_c) \quad (12)$$

Using the definition of the replacement ratio A_R given by Equation (1), Equation (12) can be rewritten as

$$q_A = \sigma_{zc} A_r + \sigma_{zs} (1 - A_r) \quad (13)$$

The stresses at the soil-column interface must be equal, thus $\sigma_{rs} = \sigma_{rc} = \sigma_r$. If kinematic relations (2), (3) and (4) are introduced in Equation (8), Equations (5), (8), (9), (10) and (13) represent a set of five equations for five unknowns: vertical stresses in the column σ_{zc} and in the soil σ_{zs} , radial stress at the soil-column interface σ_r , vertical displacement u_z and interface displacement u_r . This set of equations can be solved to obtain simple analytical closed-form solutions for displacements and stresses:

$$u_z = \frac{2H q_A}{E_{oed} C_4} \quad (14)$$

$$u_r = \frac{r_c q_A K_\psi}{E_{oed} C_4} \quad (15)$$

$$\sigma_r = \frac{q_A(C_2K_\psi + 2k_0)}{C_4} \quad (16)$$

$$\sigma_{zc} = \frac{q_A K_{pc}(C_2K_\psi + 2k_0)}{C_4} \quad (17)$$

$$\sigma_{zs} = \frac{q_A(C_1K_\psi + 2)}{C_4} \quad (18)$$

where constants K_ψ and C_4 are defined as follows:

$$K_\psi = \frac{1 + \sin \psi}{1 - \sin \psi}$$

$$C_4 = (1 - A_r)(C_1K_\psi + 2) + A_r K_{pc}(C_2K_\psi + 2k_0)$$

Constant C_4 depends only on material and geometrical properties of the column and the surrounding soil.

If the area of the applied load is sufficiently large, then the settlement of the untreated soil can be estimated as

$$u_{z,0} = \frac{q_A H}{E_{oed}} \quad (20)$$

Combining Equation (20) with Equation (14), a settlement reduction factor β , which is usually used as a measure for the improvement of the ground, can be calculated as

$$\beta = \frac{u_z}{u_{z,0}} = \frac{2}{C_4} \quad (21)$$

Stress concentration factor η defined as a ratio between vertical stresses in the soil σ_{zc} and in the column σ_{zs} can be calculated as

$$\eta = \frac{\sigma_{zc}}{\sigma_{zs}} = \frac{K_{pc}(C_2K_\psi + 2k_0)}{C_1K_\psi + 2} \quad (22)$$

Similarly, stone-column stress concentration factor η_c defined as a ratio between vertical stress in column σ_{zc} and the applied load q_A can be calculated as

$$\eta_c = \frac{\sigma_{zc}}{q_A} = \frac{K_{pc}(C_2K_\psi + 2k_0)}{C_4} \quad (23)$$

According to the above analysis the settlement reduction and the stress concentration factor depend mainly on area replacement ratio A_r , on material properties of the column material represented by the peak shear angle φ'_c , the angle of dilatancy ψ and on Poisson's ratio ν_s of the soil.

3 RESULTS AND DISCUSSION

3.1 PARAMETRIC STUDY

A parametric study has been made to show the effect of area replacement ratio A_r , peak shear strength φ'_c and especially the effect of dilation on settlement reduction and stress concentration. In stone column construction usually 15 to 35 percent of the soft soil is replaced [4]. However, the replacement ratios A_r from 5 up to 50 percent were considered in the present study.

When selecting basic input parameters of the method, such as peak and dilation angle of the column material, one should consider that the values for peak, critical and the dilation angle of the granular soil are not independent. The critical state shear angle φ'_{cv} of granular material, which is sheared under constant volume, is in general a function of mineralogy and can be treated as material property. The relationship between the peak shear angle φ'_c , the critical state shear angle φ'_{cv} and the angle of dilatancy ψ is theoretically given by Equation (7), which can be rewritten in the following form:

$$\sin \varphi'_c = \frac{\sin \varphi'_{cv} + \sin \psi}{1 + \sin \varphi'_{cv} \sin \psi} \quad (24)$$

In practice the difference between peak and critical state shear angle can also be correlated to the material's relative density and principal stress. According to the work of Bolton [12] the peak triaxial shear angle correlates to the relative density of the granular material and to the mean effective stress p' as follows:

$$\varphi'_c \approx \varphi'_{cv} + 3I_R \quad (25)$$

where I_R is a relative dilatancy index defined as

$$I_R = D_R(10 - \ln p') - 1 \quad (26)$$

The relative dilatancy index I_R can also be used for the prediction of the angle of dilatancy as proposed by Schanz and Vermeer [10]:

$$\sin \psi = \frac{I_R}{6.7 + I_R} \quad (27)$$

In the case that no laboratory data are available on the dilatancy behaviour, the strength and dilation properties of the column material can be estimated by the above relationships.

To investigate the effect of the dilation of the column material on the displacements and stresses, the angle of dilatancy equal to $\psi = 0^\circ, 5^\circ, 10^\circ, 15^\circ$ and the representative value of the critical state shear angle $\varphi'_{cv} = 35^\circ$ were adopted. Another input parameter of the method is Poisson's ratio of the soil ν_s , which was selected to be 0.35.

The results of the parametric study are presented in Figures 2, 3, 4 and 5. The effect of area replacement ratio A_r and of angle of dilatancy ψ on settlement reduction factor β is shown in Figure 2. The spacing of the columns has dominant effect on settlement reduction. As the spacing of the columns increases, the replacement ratio A_r decreases, the unit cell becomes less stiff and the total settlement increases. The settlement reduction factors are generally low and not significant for area replacement ratios lower than 4 percent ($d_c/d_c > 5$).

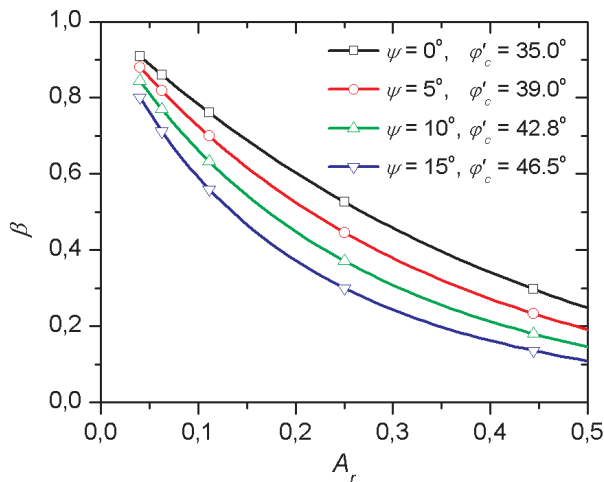


Figure 2. The effect of area replacement ratio A_r and angle of dilatancy ψ on settlement reduction factor β

The angle of dilatancy has also significant effect on settlement reduction factor β , clearly showing the importance of stone densification. High density of the column material yields high dilatancy index, hence higher dilation angle ψ could be achieved. The higher the value of dilation angle ψ , the greater is the peak shear angle φ'_c and more reduction of the settlement could be expected due to higher stress concentration in the stone-column (Fig. 2). However, the effect of the dilation of granular material at yield can not be clearly distinguished from the beneficial effect of peak shear strength of the stone column.

The effect of the dilation on settlement reduction is far more evident, if a constant value of peak shear strength is considered ($\varphi'_c = 46.5^\circ$) and different dilation properties of the granular material are taken into account. The

effect of dilation angle ψ on the settlement reduction for this case is depicted in Fig. 3. The volume increase of the granular material at yield has significant effect on the settlement reduction. For example, for the area replacement ratio A_r between 0.15 and 0.35 the total settlement for dilating stone-column ($\psi = 15^\circ$) is 16.5 to 28.0 percent lower than compared to the settlement when no dilation is taken into account.

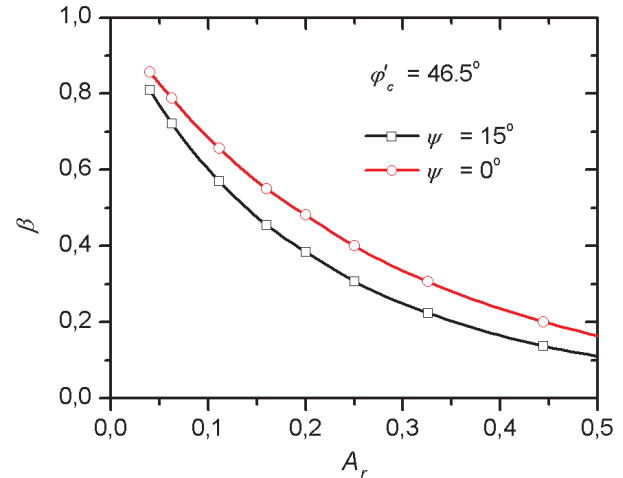


Figure 3. The effect of area replacement ratio A_r and dilation angle ψ on settlement reduction factor β

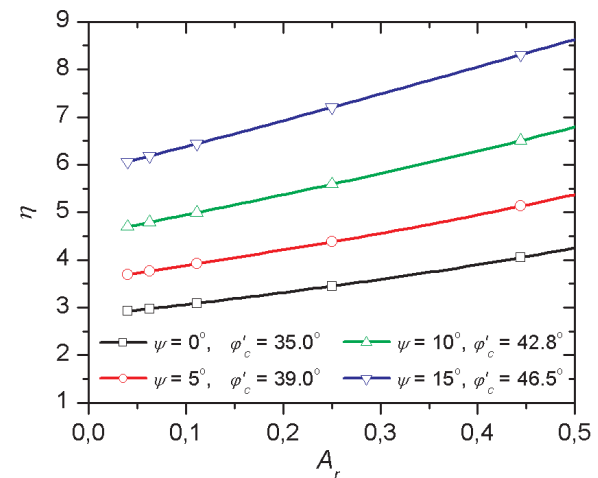


Figure 4. The effect of area replacement ratio A_r and angle of dilatancy ψ on stress concentration factor η

The effect of area replacement ratio A_r and angle of dilatancy ψ on stress concentration factor η is shown in Fig. 4. The importance of the dilation of the column material on the stress concentration in the column is clearly indicated. Well densified stone-column with high dilation angle ψ acts stiffer and can take greater proportion of the applied load. Stress concentration factors η are generally in the range from 3 to 8 for the area replacement ratios from 0.15 to 0.35.

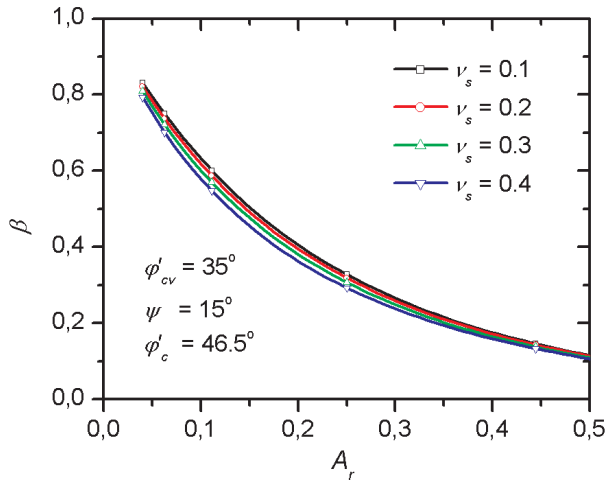


Figure 5. The effect of area replacement ratio A_r and Poisson's ratio of the soil on settlement reduction factor β

The effect of Poisson's ratio of soil is illustrated in Fig. 5, where settlement reduction factor β is depicted as a function of Poisson's ratio against the area replacement ratio A_r . The value of Poisson's ratio ν_s of soil has rather small effect on the settlement reduction, if the soil is considered under drained conditions. The effect of Poisson's ratio on the stress concentration factor η is also small.

3.2 COMPARISON WITH OTHER METHODS

It is interesting to compare the settlement reduction factors, obtained by the proposed analytical method, with some other known analytical solutions, based on similar approaches. A comparison was firstly made with elastic methods proposed by Aboshi et al. [1] and Balaam and Booker [2] and then with elastoplastic methods, which take into account the yield of the stone-column at constant volume as proposed by Priebe [6], Van Impe and De Beer [7], and finally with the method proposed by Van Impe and Madhav [8], which takes into account the yield and the dilation of the stone-column.

A direct comparison between the results is not possible because of different input parameters. However, a comparison can be made for a regular set of parameters for stone column material and for soil. The stone and soil modulus ratio $E_c/E_s = 30$, critical state shear angle $\phi'_{cv} = 35^\circ$, dilation angle $\psi = 15^\circ$ and Poisson's ratio $\nu_s = 0.3$ were adopted in the analysis. This combination of the critical shear angle and dilation angle of the column material leads to the peak shear angle $\phi'_c = 46.5^\circ$. This standard set of parameters was used in the prediction of settlements according to the above mentioned methods.

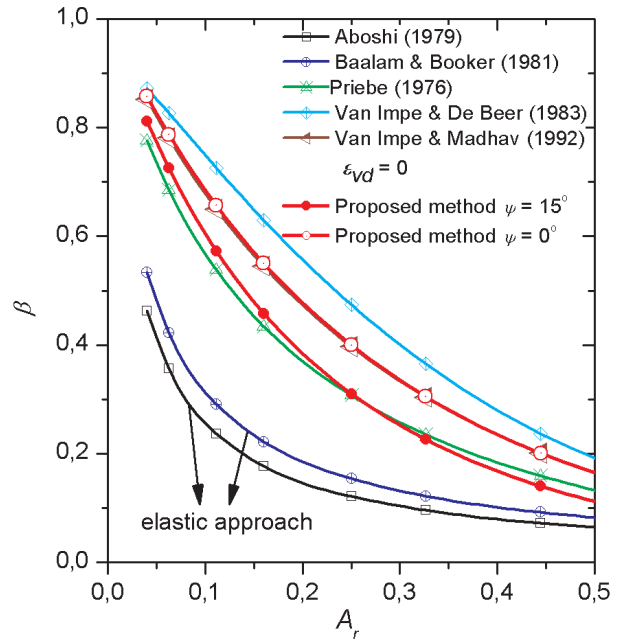


Figure 6. The comparison of settlement reduction factors according to different analytical methods

The comparison of settlement reduction factors is shown in Fig. 6. The differences in the settlement reduction factors obtained by different methods are quite significant, even if the general trend of the results is found to be similar.

The elastic methods [1, 2] give low settlement reduction factors due to supposed high stiffness of the stone column and consequently due to high stress concentration factors. If the column and soil are considered as elastic materials, then the ratio between the load carried by the column and by the soil is approximately the same as the stone and soil modulus ratio E_c/E_s . There is a simple approximate explanation why stress concentration factors are limited to a certain value. If vertical stress in the column σ_{zc} is a major principal stress in the column and vertical stress in soil, σ_{zs} is a minor principal stress in the soil and the radial stress $\sigma_r = \sigma_{rc} = \sigma_{rs}$ at soil-column interface is a minor principal stress in the column and major principal stress in the soil, then the maximum stress concentration factor at yield can be expressed as:

$$\frac{\sigma_{zc}}{\sigma_{zs}} = \frac{\sigma_{rc}}{\sigma_{rs}} = \frac{K_{pc}}{K_{ps}} = K_{pc} K_{ps} = \frac{1 + \sin \phi'_c}{1 - \sin \phi'_c} \frac{1 + \sin \phi'_s}{1 - \sin \phi'_s} = \eta_{max} \quad (28)$$

For example, peak shear angles of the granular material $\phi'_c = 45^\circ$ and soft soil $\phi'_s = 20^\circ$ give maximum stress concentration factor $\eta_{max} = 11.9$. If high

stone and soil deformation ratio E_c/E_s is considered in the elastic analysis and if the applied load q_A is high as compared to the initial (lateral) stresses in the soil, then the stress concentration factor may be too high and thus the calculated settlement underestimated. For this reason elastoplastic methods are believed to give better predictions of the settlement reduction and stress concentration factors [5].

The main difference between the proposed method and other elasto-plastic analytical methods [6-8], which also take into account column yield, is the assumption regarding the dilation of the column material. In the majority of rigid-plastic methods [6, 7] it is assumed that stone-column deforms at constant volume, thus no dilation takes place while deforming.

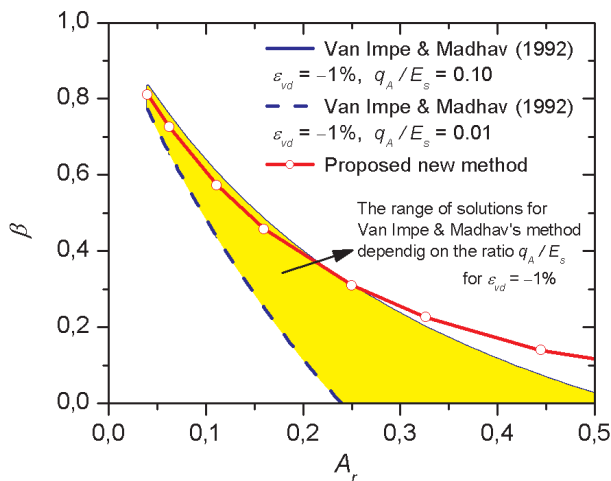


Figure 7. The comparison of settlement reduction factors for dilating stone-columns according to the proposed new and Van Impe and Madhav's method for a given set of parameters

The comparison of the results between the present and Van Impe and Madhav's method [8] is of special interest, since the dilatancy of the granular material is taken into account in both methods (Fig. 7). The present method is actually an extension of the Van Impe and Madhav's method [8] with three important differences. Small strains and vertical equilibrium of the undeformed geometry was assumed in the present method to simplify the problem and to get the closed form solution instead of nonlinear solution. These two differences have insignificant effect on the results as compared to the Van Impe and Mahav's method [8] as shown in Fig. 6, where the settlement reduction factors obtained with both methods are almost identical if no dilation is taken into account ($\epsilon_{vd} = 0$ or $\psi = 0$).

The third, and most important difference relates to the dilation. In the proposed method the dilation of the granular material at yield was considered according

to the Rowe dilatancy theory [9]. In Van Impe and Madhav's method [8] the dilation is taken into account through the final value of the volumetric strain of the column ϵ_{vd} , which must be selected in advance as an input parameter. Inappropriate selection of the volumetric strain leads to a quite large range of settlement reduction factors for different ratios between the applied load and soil elastic modulus. If inappropriate volumetric strain is selected, it can also lead to the negative settlement reduction factor β (Fig. 7). There is a simple explanation for this phenomenon. If the load level q_A/E_s is low, causing only small vertical strain, it is very unlikely that large volumetric strain $\epsilon_{vd} < 0$ would occur. If the final volumetric strain of column material $\epsilon_{vd} < 0$ is taken in advance as a constant value, then, theoretically, the dilation angle ψ of granular material becomes directly dependent on the vertical strain (Fig. 8). Especially for small vertical strains the dilation angle will be unrealistically high and will consequently lead to unrealistic prediction of settlements.

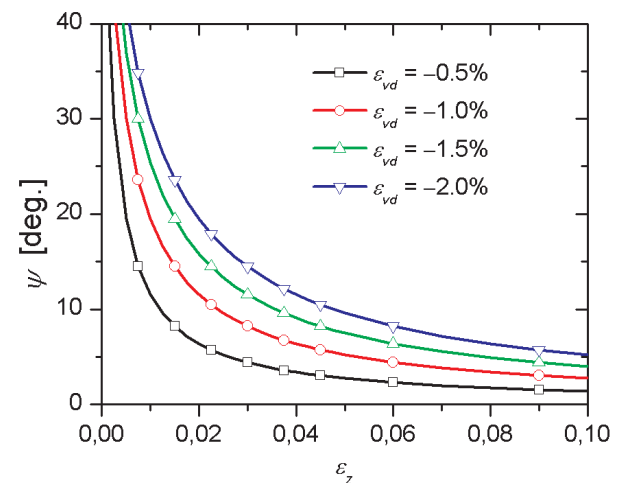


Figure 8. The effect of vertical strain ϵ_z on angle of dilatancy ψ and vertical deformation ϵ_z with the selected value of volumetric strain ϵ_{vd} according to Van Impe and Madhav's method

The correct value of volumetric strain at every level of axial strain can be determined from a triaxial test on the granular material. Nevertheless, Van Impe and Madhav's method [8] can be used in an iterative procedure. If the calculated vertical strain does not fit to the initially prescribed volumetric strain ϵ_{vd} , then the nonlinear calculation must be repeated with another value of the volumetric strain as an input parameter.

To overcome this problem, the ratio between plastic volumetric and vertical strain, defined by the dilation angle ψ according to Rowe's theory, was used in the proposed method, instead of assuming dilation volumetric strain ϵ_{vd} . In this way, the ratio between the volumetric and the vertical strain of the column at

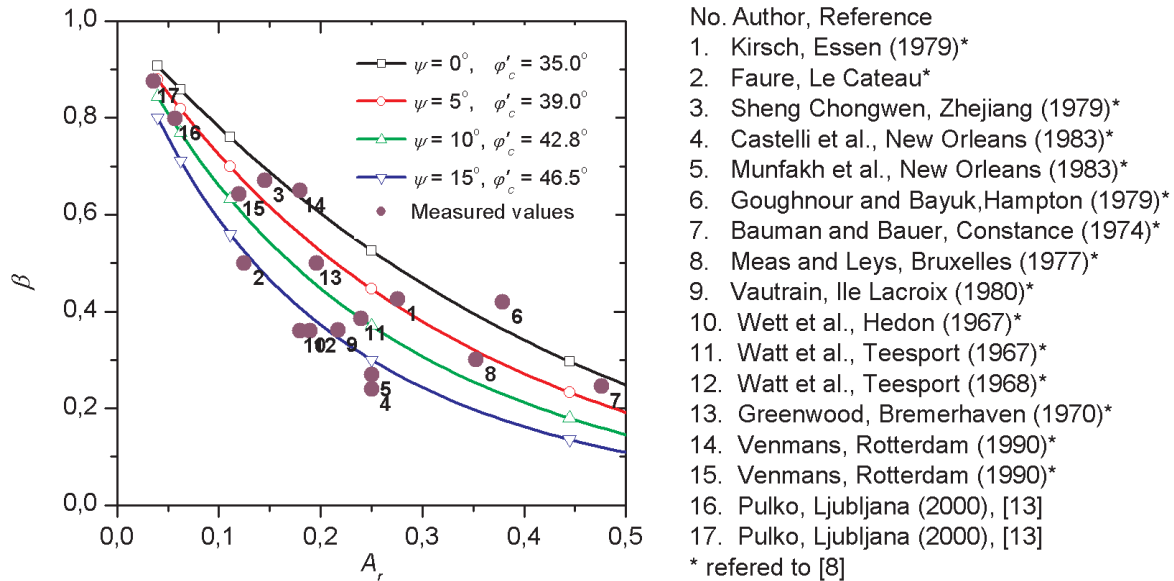


Figure 9. Comparison of settlement reduction factors β versus area replacement ratio A_r , with some field test results

yield remains constant and load independent. It should be noted, however, that the post-peak behaviour of the column material is not taken into account.

3.3 COMPARISON WITH FIELD TEST RESULTS

Figure 9 compares the predicted settlement reduction factors and corresponding area replacement ratios with some published field test results and observations. If dilation of the granular material is taken into account, then the present method is able to cover most of the field test data.

The results can not be completely conclusive due to the significant scatter of field test results and lack of well documented data in the literature. This scatter is most probably due to many factors that can affect stone-column performance, such as non-homogeneity of soil conditions, foundation shape and size, stone-column installation technique, properties of the granular material used, length of the column, different densification of the column material, different applied load, etc... Nevertheless, the predictions agree well with the field test data, thus validating the importance of the dilation in the presented method.

4 CONCLUSIONS

A simple but effective analytical method for the analysis of stone-column reinforced foundations is presented.

The stone-column and the surrounding soil are modelled as a unit cell, consisting of elastic soil and rigid plastic column material according to the Mohr-Coulomb failure law. The dilation of the column material according to the Rowe stress-dilatancy theory is directly incorporated into the method. An important feature of the method is a simple closed-form solution for the prediction of the effects of stone-columns on settlement reduction and stresses in the soil and column, which can be easily used in engineering practice.

Comparisons and some parametric analyses are presented to study the influence of area replacement ratio and material properties of the granular material on settlement reduction factor. The dilatancy of granular material has significant effect on the settlement reduction and stress concentration. Thus, densification of the column is not only important to achieve greater initial stiffness but also affects the column behaviour at yield and the overall performance of the stabilized ground.

The results are compared with some existing methods and with field test results and observations. The results of the present method agree well with most of the field data, showing the ability of the proposed analytical method to yield reasonable predictions of the behaviour of stone-column reinforced foundations. The present analytical model confirms the significance of the dilation of the column material and its effect on settlement reduction and on the stresses in the soil and column.

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APPENDIX

NOTATION

The following symbols are used in the paper:

A_c	= area of column portion;
A_e	= area of soil portion (influence area);
A_r	= replacement ratio;
C_1, C_2, C_3, C_4	= geometrical and material constants;
d_c	= diameter of stone-column;
d_e	= diameter of influence area;
D_R	= relative density;
E_{oed}	= oedometric modulus of soil;
E_c	= elastic modulus of column material;
E_s	= elastic modulus of soil;
I_R	= relative dilatancy index;
H	= column height;
k_o	= coefficient of earth pressure at rest;
K_{pc}	= passive earth pressure coefficient (column);
K_{ps}	= passive earth pressure coefficient (soil);
K_ψ	= dilation constant;
q_A	= applied vertical load;
p'	= mean effective stress;
r_c	= radius of stone-column;
r_e	= radius of influence area;
s	= stone-column spacing;
u_r	= radial displacement of stone-column;
u_z	= vertical settlement;
$u_{z,0}$	= vertical settlement of untreated soil;
β	= settlement reduction factor;
ϵ_r	= radial strain;
ϵ_{vd}	= volumetric strain of column;
ϵ_z	= vertical strain;
η	= stress concentration factor;
η_c	= column stress concentration factor;
φ'_s	= peak shear angle of soil;
φ'_c	= peak shear angle of column material;
φ'_{cv}	= shear angle of column material at critical state;
ν_s	= Poisson's ratio of soil;
$\sigma_r, \sigma_{rs}, \sigma_{rc}$	= radial stress at soil-column interface;
σ_{zc}	= vertical stress in column;
σ_{zs}	= vertical stress in soil;
ψ	= angle of dilatancy.